Integration - Area Between Curves

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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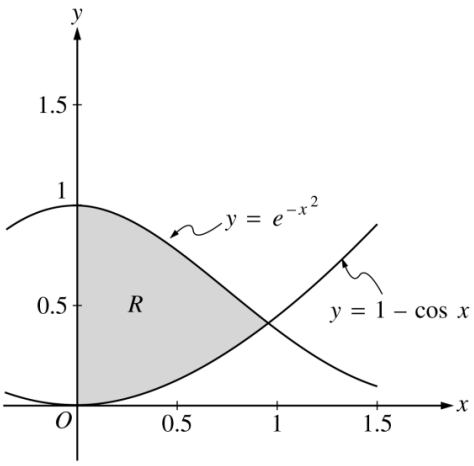
Question 1

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Volume using Cross Sections, Integration - Area Between Curves, Volume of Revolution – Washer Method, Integration Technique – Trigonometry, Integration Technique – Exponentials

Paper: Part A-Calc / Series: 2000 / Difficulty: Easy / Question Number: 1



1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

   (a) Find the area of the region R.

   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

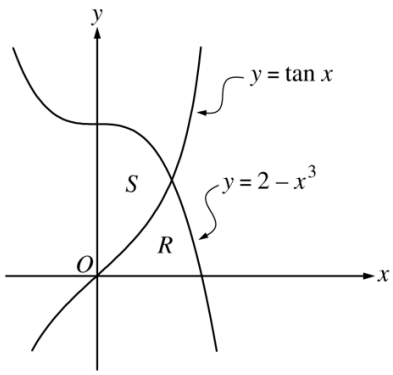
Question 2

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2001 / Difficulty: Easy / Question Number: 1



1. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

   (a) Find the area of R.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when S is revolved about the x-axis.

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Question 3

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Global or Absolute Minima and Maxima, Differentiation Technique – Exponentials

Paper: Part A-Calc / Series: 2002 / Difficulty: Somewhat Challenging / Question Number: 1

1. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

   (a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

   (b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

   (c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

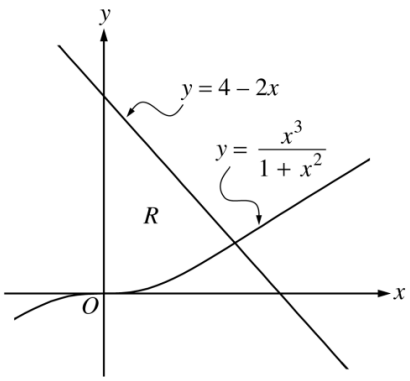
Question 4

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2002-Form-B / Difficulty: Easy / Question Number: 1



1. Let R be the region bounded by the y-axis and the graphs of $y = \frac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

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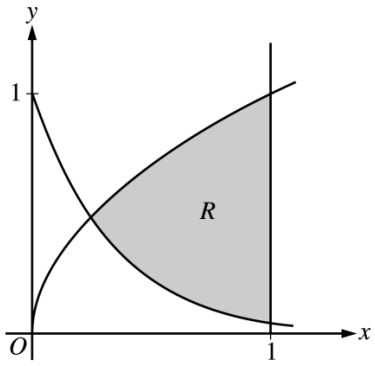
Question 5

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2003 / Difficulty: Easy / Question Number: 1



1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

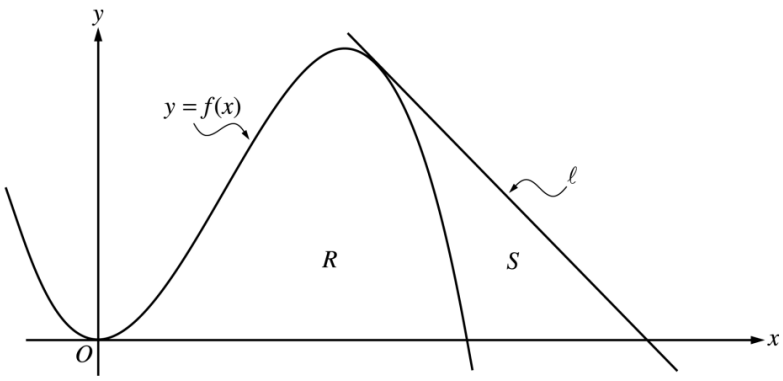
Question 6

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Tangents To Curves, Integration - Area Under A Curve, Integration - Area Between Curves, Volume of Revolution – Disc Method

Paper: Part A-Calc / Series: 2003-Form-B / Difficulty: Hard / Question Number: 1



1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

   (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when R is revolved about the x-axis.

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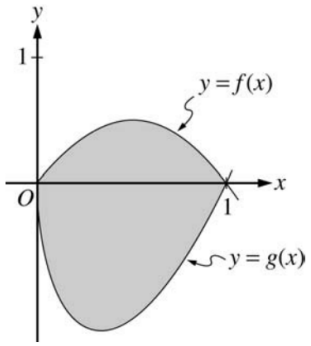
Question 7

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2004 / Difficulty: Somewhat Challenging / Question Number: 2



2. Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

   (a) Find the area of the shaded region enclosed by the graphs of f and g.

   (b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

   (c) Let h be the function given by $h(x) = kx(1 - x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

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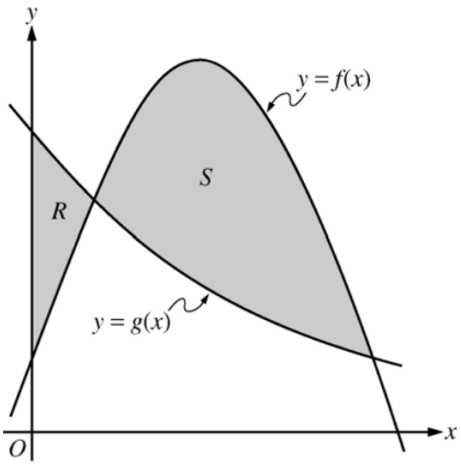
Question 8

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2005 / Difficulty: Easy / Question Number: 1



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the area of S.
   (c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

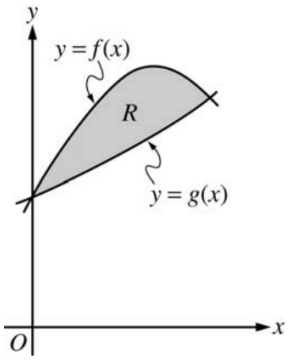
Question 9

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Volume using Cross Sections, Integration - Area Between Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2005-Form-B / Difficulty: Medium / Question Number: 1



1. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the x-axis.
   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

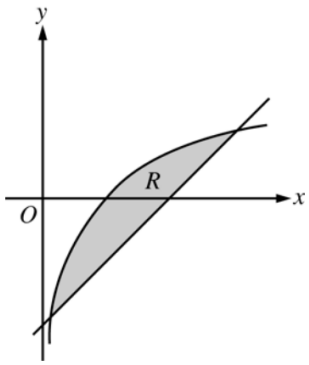
Question 10

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2006 / Difficulty: Medium / Question Number: 1



1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

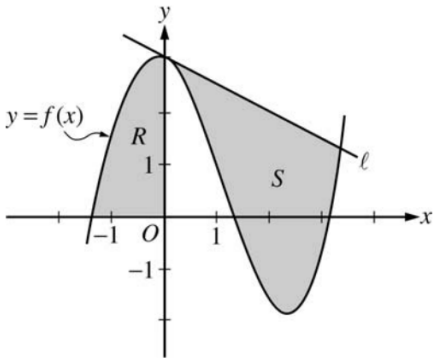
Question 11

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Integration - Area Under A Curve, Integration - Area Between Curves, Tangents To Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2006-Form-B / Difficulty: Easy / Question Number: 1



1. Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the area of S.

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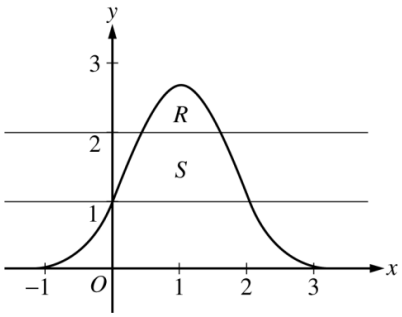
Question 12

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2007-Form-B / Difficulty: Medium / Question Number: 1



1. Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

   (a) Find the area of R.

   (b) Find the area of S.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

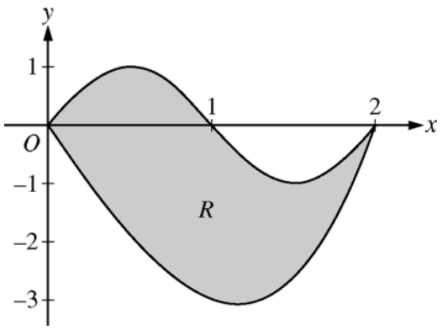
Question 13

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume using Cross Sections

Paper: Part A-Calc / Series: 2008 / Difficulty: Medium / Question Number: 1



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

   (a) Find the area of R.

   (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

   (d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

Question 14

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2008-Form-B / Difficulty: Easy / Question Number: 1

1. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

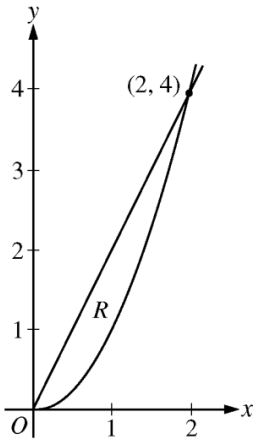
Question 15

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume using Cross Sections, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2009 / Difficulty: Easy / Question Number: 4



4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

   (c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

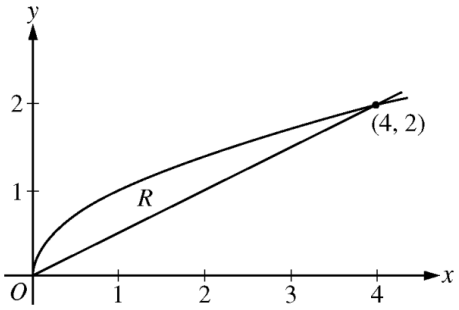
Question 16

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume using Cross Sections, Volume of Revolution – Washer Method, Integration Technique - Harder Powers

Paper: Part B-Non-Calc / Series: 2009-Form-B / Difficulty: Easy / Question Number: 4



4. Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

   (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

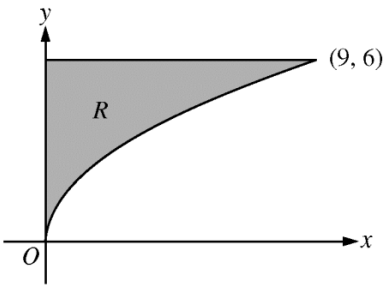
Question 17

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part B-Non-Calc / Series: 2010 / Difficulty: Easy / Question Number: 4



4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

   (a) Find the area of R.

   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

   (c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

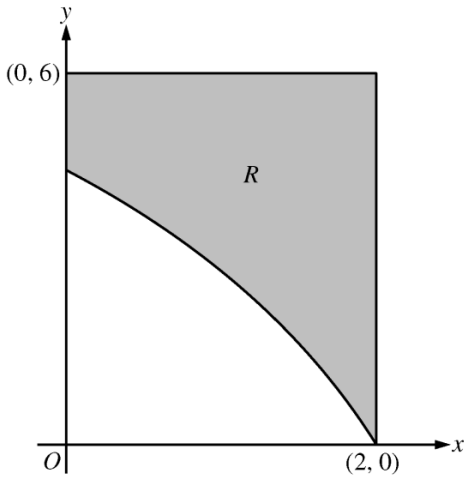
Question 18

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2010-Form-B / Difficulty: Easy / Question Number: 1



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

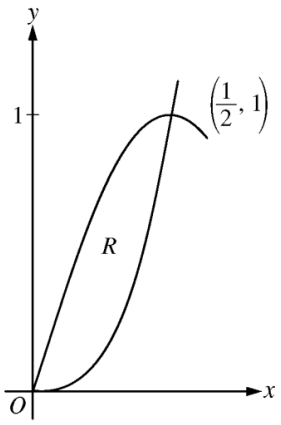
Question 19

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration, Integration

Subtopics: Tangents To Curves, Integration - Area Between Curves, Integration Technique – Trigonometry, Volume of Revolution – Washer Method

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Easy / Question Number: 3



3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

   (a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

   (b) Find the area of R.

   (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

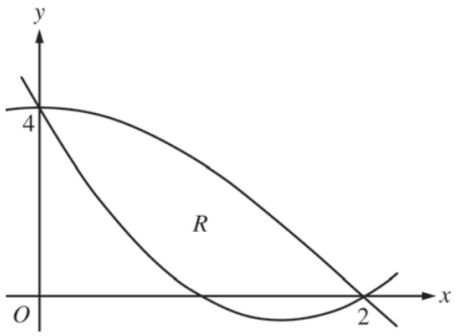
Question 20

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Medium / Question Number: 5



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

   (a) Find the area of R.

   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

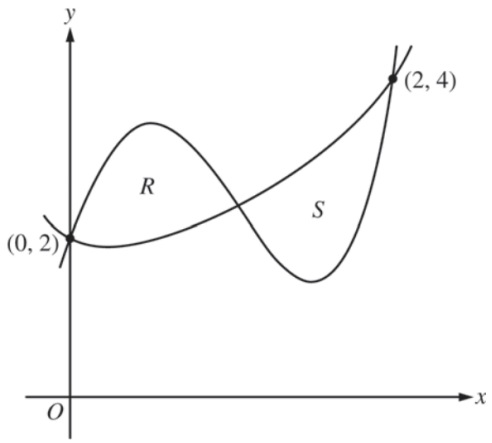
Question 21

Qualification: AP Calculus AB

Areas: Applications of Integration, Differentiation

Subtopics: Integration - Area Between Curves, Volume using Cross Sections, Rates of Change (Instantaneous)

Paper: Part A-Calc / Series: 2015 / Difficulty: Medium / Question Number: 2



2. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

   (a) Find the sum of the areas of regions R and S.

   (b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

   (c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

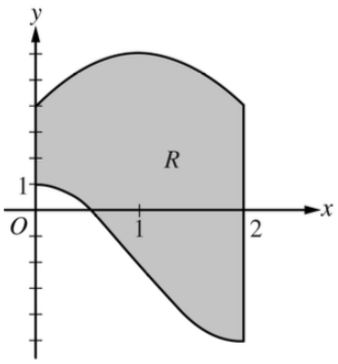
Question 22

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume using Cross Sections, Volume of Revolution – Washer Method, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Medium / Question Number: 5



5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

   (a) Find the area of R.

   (b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

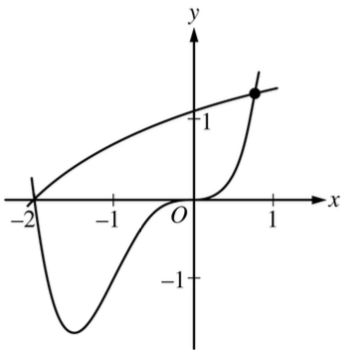
Question 23

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Integration - Area Between Curves, Increasing/Decreasing , Volume using Cross Sections, Rates of Change (Instantaneous), Differentiation Technique – Chain Rule

Paper: Part A-Calc / Series: 2022 / Difficulty: Medium / Question Number: 2



2. Let f and g be the functions defined by $f(x) = \ln(x+3)$ and $g(x) = x^4 + 2x^3$. The graphs of f and g, shown in the figure above, intersect at $x = -2$ and $x = B$, where $B > 0$.

   (a) Find the area of the region enclosed by the graphs of f and g.

   (b) For $-2 \le x \le B$, let $h(x)$ be the vertical distance between the graphs of f and g. Is h increasing or decreasing at $x = -0.5$? Give a reason for your answer.

   (c) The region enclosed by the graphs of f and g is the base of a solid. Cross sections of the solid taken perpendicular to the x-axis are squares. Find the volume of the solid.

   (d) A vertical line in the xy-plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

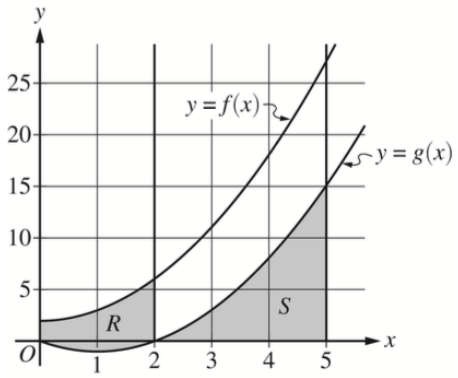
Question 24

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume using Cross Sections, Volume of Revolution – Washer Method

Paper: Part B-Non-Calc / Series: 2024 / Difficulty: Medium / Question Number: 6



6. The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

   (a) Let R be the region bounded by the graphs of f and g, from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R.

   (b) Let S be the region bounded by the graph of g and the x-axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height equal to half its base in region S. Find the volume of the solid. Show the work that leads to your answer.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.